

Mathematics*Time Allowed: 3 hours**Maximum : 90***General Instructions:**

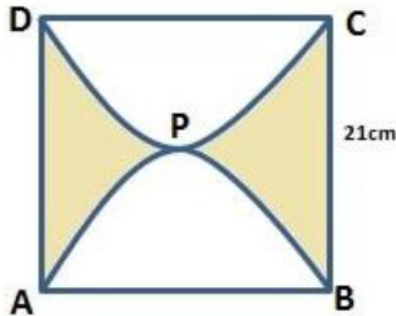
1. *All questions are compulsory.*
2. *The question paper consists of 31 questions divided into four sections A, B, C and D.*
3. *Section A contains 4 questions of 1 mark each. Section B contains 6 questions of 2 marks each. Section C contains 10 questions of 3 marks each. Section D contains 11 questions of 4 marks each.*
4. *Use of calculators is not permitted.*

SECTION A

- 1 Find the value of k, for which one root of the quadratic equation $kx^2 - 14x + 8 = 0$ is six times the other. 1
- 2 Find the tenth term of the sequence $\sqrt{2}, \sqrt{8}, \sqrt{18}, \dots$ 1
- 3 Out of 200 bulbs in a box, 12 bulbs are defective. One bulb is taken out at random from the box. What is the probability that the drawn bulb is not defective? 1
- 4 Find the value of a, for which point P $(\frac{a}{3}, 2)$ is the midpoint of the line segment joining the points Q (-5, 4) and R(-1, 0). 1

SECTION B

- 5 If 2 is a root of the equation $x^2+kx+12=0$ and the equation $x^2+kx+q=0$ has equal roots, find the value of q . 2
- 6 How many two digit numbers are divisible by 7? 2
- 7 Find a relation between x & y such that the point $P(x,y)$ is equidistant from the points $A(-5,3)$ and $B(7,2)$. 2
- 8 Find the perimeter of the shaded region if $ABCD$ is a square of side 21 cm and APB & CPD are semicircles. (Use $\pi=22/7$). 2



- 9 A sphere of diameter 6cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel? 2
- 10 Find the number of coins of 1.5 cm diameter and 0.2 cm thickness to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm. 2

SECTION C

11 Solve $\frac{1}{(a+b+x)} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$, $a+b \neq 0$. 3

12 3

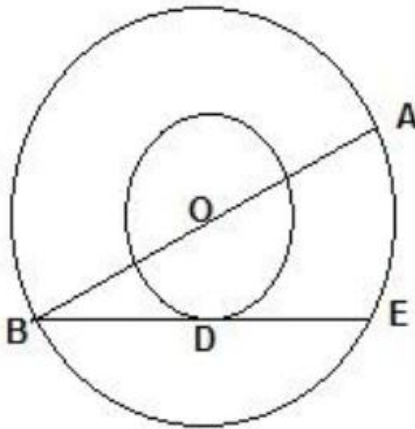
In an AP, the sum of first n terms is

$$\frac{3n^2}{2} + \frac{13n}{2}$$

Find the 25th term.

13 The ninth term of an AP is equal to seven times the second term and 3
twelfth term exceeds five times the third term by 2. Find the first
term and the common difference.

14 In the given figure, the radii of two concentric circles are 13 cm and 8 3
cm. AB is diameter of the bigger circle. BD is the tangent to the
smaller circle touching it at D. Find the length AD.



15 P & Q are centres of circles of radii 9 cm and 2 cm respectively. $PQ = 17$ cm. 3
R is the centre of the circle of radius x cm which touches the
above circle externally. Given that angle PRQ is 90° . Write an
equation in x and solve it.

16 Draw a triangle ABC in which $AB = 5$ cm, $BC = 6$ cm and angle ABC 3

$=60^\circ$. Construct a triangle whose sides are $\frac{5}{7}$ times the corresponding sides of triangle ABC.

- 17 One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting (a) Non face card, (b) Black king or a Red queen, (c) Spade card. **3**
- 18 Find the area of shaded region shown in the given figure where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre. **3**



- 19 Water is flowing at the rate of 0.7 m/sec through a circular pipe whose internal diameter is 2 cm into a cylindrical tank, the radius of whose base is 40 cm. Determine the increase in the level of water in half hour. **3**
- 20 The perimeters of the ends of the frustum of a cone are 207.24 cm and 169.56 cm. If the height of the frustum be 8 cm, find the whole surface area of the frustum. (Use $\pi = 3.14$) **3**

SECTION D

- 21** Three eighth of the students of a class opted for visiting an old age home. Sixteen students opted for having a nature walk. Square root of total number of students in the class opted for tree plantation in the school. The number of students who visited an old age home is same as the number of students who went for a nature walk and did tree plantation. Find the total number of student. 4
What values are inculcated in students through such activities?
- 22** The minimum age of children to be eligible to participate in a painting competition is 8 years. It is observed that the age of youngest boy was 8 years and the ages of rest of participants are having a common difference of 4 months.If the sum of ages of all the participants is 168 years, find the age of eldest participant in the painting competition. 4
- 23** Prove that the lengths of the tangents drawn from an external point to a circle are equal. Using the above theorem, prove that $AB+CD = AD+BC$, if a quadrilateral ABCD is drawn to circumscribe a circle. 4
- 24** Draw a pair of tangents inclined to each other at an angle of 60° to a circle of radius 3 cm. 4
- 25** From the top of tower, 100 m high, a man observes two cars on the opposite sides of the tower with the angles of depression 30° & 45° respectively. Find the distance between the cars. (Use $\sqrt{3}=1.73$). 4
- 26** From a point on the ground, the angles of elevation of the bottom and top of a tower fixed at the top of a 20 m high building are 45° & 60° respectively. Find the height of the tower. 4

- 27** Cards marked with numbers 3, 4, 5,, 50 are placed in a bag and mixed thoroughly. One card is drawn at random from the bag. Find the probability that number on the card drawn is: 4
- a. Divisible by 7.
- b. A perfect square
- c. A multiple of 6
- .
- 28** If P(9a-2, -b) divides the line segment joining A(3a+1, -3) and B(8a, 5) in the ratio 3:1. Find the values of a & b. 4
- 29** Find the coordinates of the points which divide the line segment joining A(2, -3) and B(-4, -6) into three equal parts. 4
- 30** Four equal circles are described at the four corners of a square so that each touches two of the others. The shaded area enclosed between the circles is 4
- $$\frac{24}{7}\text{cm}^2.$$
- Find the radius of each circle.
- 31** A right triangle having sides 15 cm and 20 cm is made to revolve about its hypotenuse. Find the Volume and Surface Area of the double cone so formed. (Use $\pi=3.14$). 4

Answer Key

SECTION A

1 $K = 3$

2 $\sqrt{200}$

3 $\frac{47}{50}$

4 $a = -9$

5 2 is the root of $x^2 + kx + 12 = 0$

$$\Rightarrow (2)^2 + 2k + 12 = 0$$

$$\Rightarrow 2k + 16 = 0$$

$$k = -8$$

Put $k = -8$ in $x^2 + kx + q = 0$

$$\Rightarrow x^2 - 8x + q = 0$$

For equal roots

$$(-8)^2 - 4(1)q = 0$$

$$64 - 4q = 0$$

$$4q = 64$$

$$q = 16$$

6

Two digit numbers which are divisible by 7 are

14,21,28,.....98.

It forms an A.P.

$$a=14, d=7, a_n=98$$

$$a_n=a+(n-1) d$$

$$98=14+(n-1)7$$

$$98-14=7n-7$$

$$84+7=7n$$

$$7n=91$$

$$n=13$$

7

Let P(x,y) is equidistant from A(-5,3) and B(7,2)

$$AP=BP$$

$$\Rightarrow \sqrt{(x+5)^2+(y-3)^2} = \sqrt{(x-7)^2+(y-2)^2}$$

$$\Rightarrow x^2+10x+25+y^2-6y+9 = x^2-14x+49+y^2-4y+4$$

$$10x-6y+34 = -14x-4y+53$$

$$10x+14x-6y+4y = 53-34$$

$$24x-2y = 19$$

$24x - 2y - 19 = 0$ is the required relation

8

Perimeter of the shaded region

$$= AD+ BC + \text{lengths of semi circles APB\&CPD}$$

$$= 21+21+2\left(2x\frac{22}{7}x\frac{21}{2}\right)$$

$$= 42+2(66)$$

$$= 42+ 132$$

$$=174 \text{ cm}$$

9 Let the water level raised in cylindrical vessel be h cm

Volume of Sphere = Volume of water displaced in cylinder

$$\frac{4}{3}\pi (3)^3 = \pi (6)^2 h$$

$$\frac{4}{3} \times 27 = 36 h$$

$$36 = 36h$$

$$h = 1 \text{ cm}$$

10 Volume of Coin = $\pi r^2 h$

$$= \frac{22}{7} \times (0.75)^2 \times 0.2 \text{ cm}^3$$

$$\text{Volume of Cylinder} = \frac{22}{7} \times (2.25)^2 \times 10 \text{ cm}^3$$

No. of Coins = Volume of Cylinder / Volume of Coin

$$= \left(\frac{22}{7} \times (2.25)^2 \times 10 \right) / \left(\frac{22}{7} \times (0.75)^2 \times 0.2 \right)$$

$$= 450$$

11

$$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$$

$$\Rightarrow \frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$

$$\Rightarrow \frac{x-(a+b+x)}{x(a+b+x)} = \frac{a+b}{ab}$$

$$\Rightarrow \frac{x-a-b-x}{x(a+b+x)} = \frac{a+b}{ab}$$

$$\Rightarrow \frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab}$$

$$\Rightarrow x(a+b+x) = -ab$$

$$\Rightarrow x^2 + (a+b)x - ab = 0$$

$$\Rightarrow (x+a)(x+b) = 0$$

$$\Rightarrow x = -a \text{ or } x = -b$$

12

$$S_n = \frac{3n^2 + 13n}{2}$$

$$a_n = S_n - S_{n-1}$$

$$\Rightarrow a_{25} = S_{25} - S_{24}$$

$$= \frac{3(25)^2 + 13(25)}{2} - \frac{3(24)^2 + 13(24)}{2}$$

$$= \frac{1}{2} \{ 3(25^2 - 24^2) + 13(25 - 24) \}$$

$$= \frac{1}{2} (3 \times 49 + 13) = 80$$

13

Let the first term of A.P be a and common difference be d .

$$a_9 = 7a_2$$

$$\Rightarrow a + 8d = 7(a + d) \quad \dots\dots\dots (1)$$

$$a_{12} = 5a_3 + 2$$

$$\Rightarrow a + 11d = 5(a + 2d) + 2 \quad \dots\dots\dots (2)$$

From (1), $a + 8d = 7a + 7d$

$$-6a + d = 0 \quad \dots\dots\dots (3)$$

From (2), $a + 11d = 5a + 10d + 2$

$$-4a + d = 2 \quad \dots\dots\dots (4)$$

Subtracting (4) from (3)

$$-2a = -2$$

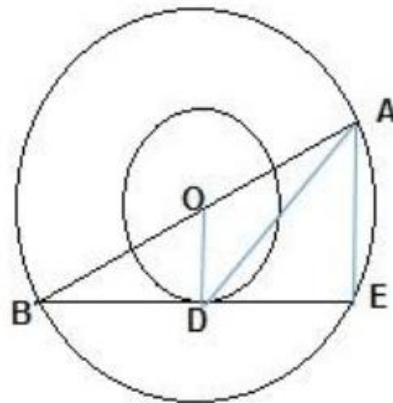
$$\Rightarrow a = 1$$

From (3),

$$-6 + d = 0$$

$$d = 6$$

14



Join OD and AE

$\angle ODB = 90^\circ$ (radius is perpendicular to tangent at point of contact)

$\angle AEB = 90^\circ$ (angle in a semicircle)

$OD \parallel AE$ (Corresponding Angles)

$$AE = 2 \times OD$$

$$= 2 \times 8 = 16 \text{ cm}$$

In right $\triangle ODB$, $BD^2 = 13^2 - 8^2$

$$= 169 - 64 = 105$$

$$BD = \sqrt{105} \text{ cm}$$

$$DE = \sqrt{105} \text{ cm}$$

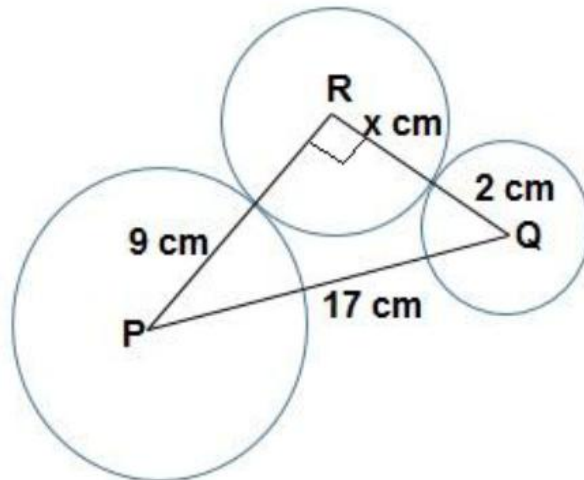
In right $\triangle AED$, $AD^2 = AE^2 + DE^2$

$$= 16^2 + (\sqrt{105})^2$$

$$= 256 + 105 = 361$$

$$AD = 19 \text{ cm}$$

15



In right $\triangle PQR$, by Pythagoras theorem

$$PQ^2 = PR^2 + RQ^2$$

$$\begin{aligned} \Rightarrow 17^2 &= (x+9)^2 + (x+2)^2 \\ \Rightarrow x^2 + 11x - 102 &= 0 \\ \Rightarrow x^2 + 17x - 6x - 102 &= 0 \\ \Rightarrow x(x+17) - 6(x+17) &= 0 \\ \Rightarrow (x-6)(x+17) &= 0 \\ \Rightarrow x = 6 \text{ or } x = -17 \\ \Rightarrow x = 6 \text{ cm (x can't be negative)} \end{aligned}$$

16 For correct construction

17 Total number of cards = 52
 Number of non face cards = $52 - 12$
 = 40

$$P(\text{ non-face cards}) = \frac{40}{52} = \frac{10}{13}$$

Number of black kings = 2

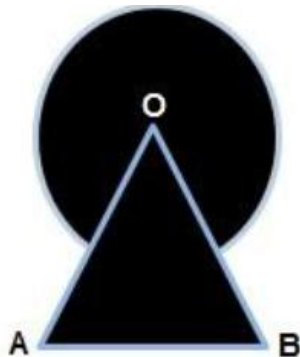
Number of red queens = 2

$$P(\text{ a black King or a red queen}) = \frac{4}{52}$$

Number of spade cards = 13

$$P(\text{ Spade cards}) = \frac{13}{52}$$

18



$$\angle AOB = 60^\circ$$

Area of shaded region

= Area of $\triangle AOB$ + Area of major sector of circle

$$= \frac{\sqrt{3}}{4} (12)^2 + \frac{300^\circ}{360^\circ} \times \frac{22}{7} \times (6)^2 \text{ cm}^2$$

$$= 36\sqrt{3} + \frac{660}{7} \text{ cm}^2$$

19 Length of water that flows out in 30 minutes

$$= (0.7 \times 100 \times 60 \times 30) \text{ cm}$$

$$= 126000 \text{ cm}$$

Volume of water that flows out in 30 minutes

$$= \pi (1)^2 \times 126000 \text{ cm}^3$$

$$= 126000 \pi \text{ cm}^3$$

Let the depth of water in the tank be x cm

Volume of water in tank

$$= \pi (40)^2 \times x \text{ cm}^3$$

According to the question

$$\pi (40)^2 \times x = 126000\pi$$

$$\Rightarrow x = 78.75 \text{ cm}$$

20

Let R and r be the radii of the circular ends of the frustum. ($R > r$)

$$2\pi R = 207.24$$

$$R = 207.24 / (2 \times 3.14)$$

$$R = 33 \text{ cm}$$

$$2\pi r = 169.56 \text{ cm}$$

$$r = 169.56 / (2 \times 3.14)$$

$$r = 27 \text{ cm}$$

$$l^2 = h^2 + (R-r)^2$$

$$= 8^2 + (33-27)^2$$

$$l = 10 \text{ cm}$$

Whole surface area of the frustum

$$= \pi (R^2 + r^2 + (R+r)l)$$

$$= 3.14 ((33)^2 + (27)^2 + (33+27)10)$$

$$= 3.14 (1089 + 729 + 600)$$

$$= 3.14 \times 2418 \text{ cm}^2$$

$$= 7592.52 \text{ cm}^2 [1]$$

21

Let the total number of students be x

$$\frac{3}{8}x = 16 + \sqrt{x}$$

$$\Rightarrow \frac{3}{8}x - 16 = \sqrt{x}$$

$$\Rightarrow 3x - 128 = 8\sqrt{x}$$

$$\Rightarrow 3x - 8\sqrt{x} - 128 = 0$$

Let $\sqrt{x} = y$

$$3y^2 - 8y - 128 = 0$$

$$\Rightarrow 3y^2 - 24y + 16y - 128 = 0$$

$$\Rightarrow 3y(y-8) + 16(y-8) = 0$$

$$\Rightarrow (y-8)(3y+16) = 0$$

$$y = 8 \text{ or } y = -16/3$$

$$y = 8 \Rightarrow x = 64$$

$$y = -16/3 \Rightarrow x = 256/9$$

number of students = 64

Values inculcated

22 $a = 8, d = 1/3 \text{ years}, S_n = 168$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\Rightarrow 168 = \frac{n}{2} [2(8) + (n-1)\frac{1}{3}]$$

$$n^2 + 47n - 1008 = 0$$

$$\Rightarrow n^2 + 63n - 16n - 1008 = 0$$

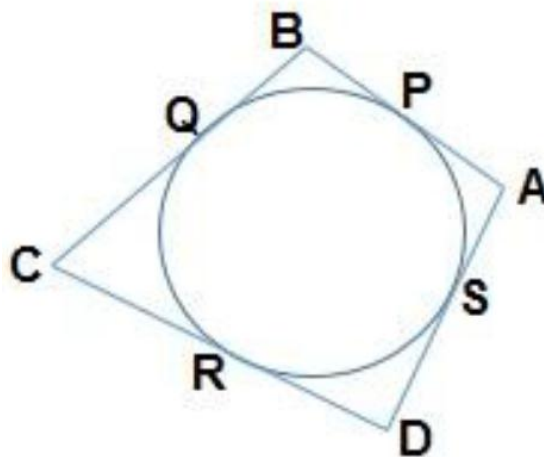
$$\Rightarrow (n-16)(n+63) = 0$$

$$\Rightarrow n = 16 \text{ or } n = -63$$

$$n = 16 \quad (n \text{ cannot be negative})$$

Age of the eldest participant = $a + 15d = 13$ years

23 Correct Proof of the theorem



In the given figure,

Using the above theorem

$$AP = AS \dots\dots\dots(1)$$

$$BP = BQ \dots\dots\dots(2)$$

$$DR = DS \dots\dots\dots(3)$$

$$CR = CQ \dots\dots\dots(4)$$

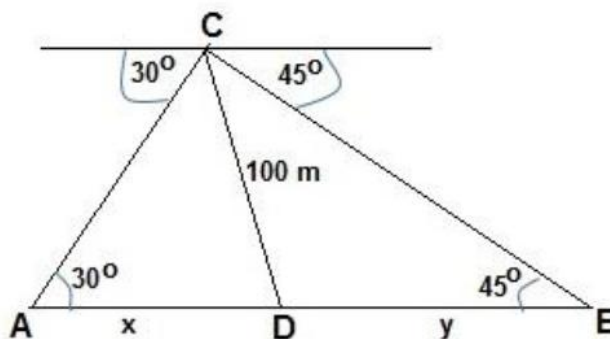
Adding (1), (2),(3) and (4), we get

$$(AB+BP) + (DR+CR) = (AS+DS) + (BQ+CQ)$$

$$\Rightarrow AB + CD = AD + BC$$

24 For correct constructions

25



Correct diagram

In right $\triangle ADC$

$$\tan 30^\circ = \frac{CD}{AD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{x}$$

$$\Rightarrow x = 100\sqrt{3} \dots\dots\dots (1)$$

In right $\triangle BDC$

$$\tan 45^\circ = \frac{CD}{DB}$$

$$\Rightarrow 1 = \frac{100}{y}$$

$$\Rightarrow y = 100 \text{ m}$$

Distance between two cars

$$= AB = AD + DB$$

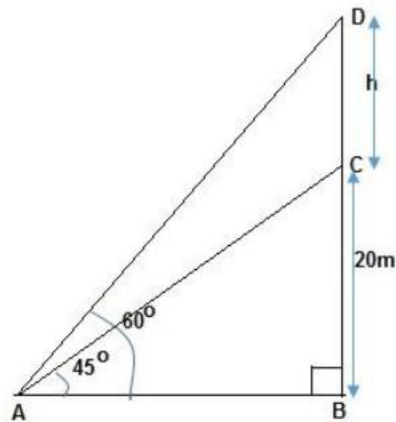
$$= (100\sqrt{3} + 100)\text{m}$$

$$= (100 \times 1.73 + 100)\text{m}$$

$$= (173 + 100) \text{ m}$$

$$= 273 \text{ m}$$

26



Let BC be building of height 20m and CD be the tower of height h m.
 Let A be point on the ground at a distance of x m from the foot of the building.

In right $\triangle ABC$,

$$\tan 45^\circ = \frac{BC}{AB}$$

$$\Rightarrow 1 = \frac{20}{x}$$

$$\Rightarrow x = 20\text{m} \dots\dots\dots(1)$$

In right $\triangle ABD$,

$$\tan 60^\circ = \frac{BD}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{h+20}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h+20}{20}$$

$$\begin{aligned} \Rightarrow h &= 20\sqrt{3} - 20 \\ &= 20(\sqrt{3} - 1) \\ &= 20 \times 0.732 \\ &= 14.64 \text{ m} \end{aligned}$$

Height of tower = 14.64 m

27

Total number of cards = 48

Probability of an event = $\frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}}$

Number of cards divisible by 7 = 7

$$P(\text{cards divisible by 7}) = \frac{7}{48}$$

Number of cards having a perfect square = 6

$$P(\text{cards having a perfect square}) = \frac{6}{48} = \frac{1}{8}$$

Number of multiples of 6 from 3 to 50 = 8

$$P(\text{multiple of 6 from 3 to 50}) = \frac{8}{48} = \frac{1}{6}$$

28

By Section formula

$$9a-2 = \frac{3(8a)+1(3a+1)}{3+1} \dots\dots\dots(1)$$

$$-b = \frac{3(5)+1(-3)}{3+1} \dots\dots\dots(2)$$

From (2)

$$-b = \frac{15-3}{4} = 3$$

$$b = -3$$

From (1)

$$9a - 2 = \frac{24a+3a+1}{4}$$

$$4(9a - 2) = 27a + 1$$

$$36a - 8 = 27a + 1$$

$$9a = 9$$

$$a = 1$$

29

Let P(x₁, y₁) and Q(x₂, y₂) are two points which divide AB in three equal parts.

By Section formula

$$P(x_1, y_1) = \left(\frac{1X(-4)+2X(2)}{1+2}, \frac{1X(-6)+2X(-3)}{1+2} \right)$$

$$= \left(\frac{-4+4}{3}, \frac{-6+(-6)}{3} \right)$$

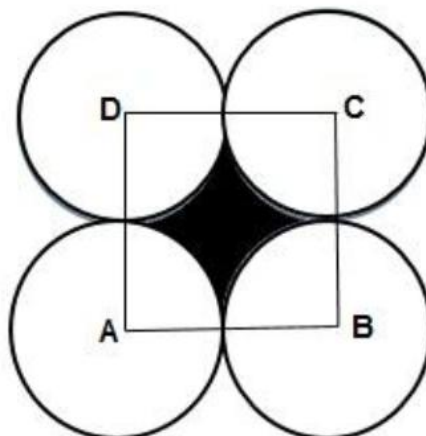
$$= (0, -4)$$

$$Q(x_2, y_2) = \left(\frac{2X(-4)+1X(2)}{2+1}, \frac{2X(-6)+1X(-3)}{2+1} \right)$$

$$= \left(\frac{-8+2}{3}, \frac{-12+(-3)}{3} \right)$$

$$= (-2, -5)$$

30



Let r cm be the radius of each circle.

$$\text{Area of square} - \text{Area of 4 sectors} = \frac{24}{7} \text{ cm}^2$$

$$\Rightarrow (2r)^2 - 4 \left(\frac{90^\circ}{360^\circ} \times \pi r^2 \right) = \frac{24}{7}$$

$$\Rightarrow 4r^2 - \frac{22}{7}r^2 = \frac{24}{7}$$

$$\Rightarrow \frac{28r^2 - 22r^2}{7} = \frac{24}{7}$$

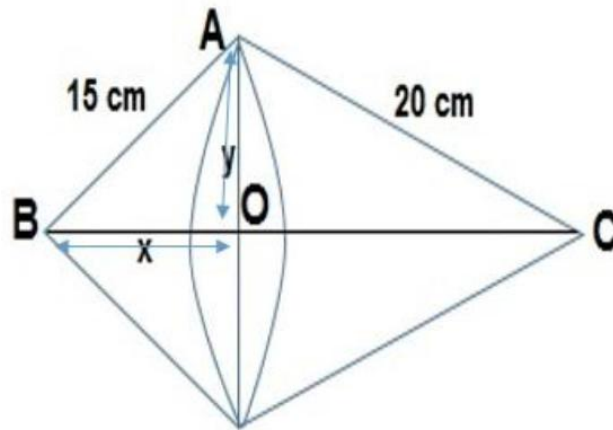
$$\Rightarrow 6r^2 = 24$$

$$\Rightarrow r^2 = 4$$

$$\Rightarrow r = \pm 2$$

\Rightarrow radius of each circle is 2 cm (r cannot be negative)

31



In right Δ BAC, by Pythagoras theorem

$$\begin{aligned}BC^2 &= AB^2 + AC^2 \\ &= 15^2 + 20^2 \\ &= 225 + 400 \\ &= 625\end{aligned}$$

$$BC = 25\text{cm}$$

Let OA = y cm and OB = x cm

$$x^2 + y^2 = 15^2$$

$$(25-x)^2 + y^2 = 20^2$$

Solving we get x=9 and y=12

\therefore OA = 12 cm and OB = 9 cm

$$\begin{aligned}\text{Volume of double cone} &= \frac{1}{3}\pi (OA)^2 \times OC + \frac{1}{3}\pi (OA)^2 \times OB \\ &= \frac{1}{3} \times 3.14 \times (12)^2 \times (OC + OB) \\ &= \frac{1}{3} \times 3.14 \times 144 \times 25 \\ &= 3768 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\text{Surface area of double cone} &= \pi \times OA \times AC + \pi \times OA \times AB \\ &= \pi \times 12 \times 20 + \pi \times 12 \times 15 \\ &= 420 \pi \text{ cm}^2 \\ &= 1318.8 \text{ cm}^2\end{aligned}$$